Yesterday, we introduced Bardeen variables as a way to avoid the gauge problem.
Another, alternative, approach is to 'fix the gauge and keep track of all perturbations'.
Some popular gauges are:

- Newtonian gauge - freedom in T and L $\left(\begin{cases} \xi^0 = T \\ \xi^i = L^i = \partial^i L + \hat{L}^i \end{cases}\right)$ allows us to set two of the four scalar perturbations to 0.
  Thus we define the gauge by $\boxed{B = E = 0}$

- Spatially flat gauge - Convenient gauge for computing inflationary perturbations.
  Defined by $\boxed{C = E = 0}$

- Synchronous gauge - Historically significant, but can cause unecessary confusion
  Defined by $\boxed{A = B = 0}$

Picking the Newtonian gauge we can rewrite the perturbed metric as

$$
\begin{aligned}
ds^2 &= a^2(\eta)\left[-(1+2A)\,d\eta^2 + 2\cancelto{0}{(B_i)}\,dx^i\,d\eta + \left(\delta_{ij} + 2(C\delta_{ij} + \partial_{\langle i}\partial_{j\rangle}\cancelto{0}{E})\right)dx^i\,dx^j\right] \\
&= a^2(\eta)\left[-(1+2A)\,d\eta^2 + \delta_{ij}(1+2C)\,dx^i\,dx^j\right] \\
&:= a^2(\eta)\left[-(1+2\Psi)\,d\eta^2 + (1-2\Phi)\delta_{ij}\,dx^i\,dx^j\right] \qquad \text{where} \begin{cases} A \equiv \Psi \\ C \equiv -\Phi \end{cases}
\end{aligned}
$$

Note that here we have used the fact that at linear order it is consistent to set the vector and tensor perturbations to 0.

Helpfully, in this gauge the metric is diagonal, thus:

- hypersurfaces of constant time are orthogonal to the worldlines of observers at rest in the coordinates ($B = 0$)
- the induced geometry of the constant-time surfaces is isotropic ($E = 0$)

This gauge will be the preferred choice for studying structure formation.

P.T.O

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6.1.2 - Matter Perturbations

~ Perturbations of the energy-momentum tensor

For ease we use the following:

$$\begin{cases} T^0{}_0 \equiv -(\bar{\rho} + \delta\rho) \\ T^i{}_0 \equiv -(\bar{\rho} + \bar{P}) v^i \\ T^i{}_j \equiv (\bar{P} + \delta P)\delta^i_j + \Pi^i{}_j \qquad \Pi^i{}_i = 0 \end{cases}$$

v^i - bulk velocity

$\Pi^i{}_j$ - anisotropic stress

Def$^{\underline{n}}$. (momentum density) $\quad q^i \equiv (\bar{\rho} + \bar{P}) v^i$

The total $T_{\mu\nu}$ is built additively from the components: $\quad T_{\mu\nu} = \sum_a T^{(a)}_{\mu\nu}$

Thus,

$$\delta\rho = \sum_a \delta\rho_a \qquad \delta P = \sum_a \delta P_a \qquad q^i = \sum_a q^i_{(a)} \qquad \Pi^{ij} = \sum_a \Pi^{ij}{}_{(a)}$$

velocity perturbations do not add but momentum densities do.